BCME - Research Presentation

Learning fractions through visual representations - a Ph.D. research with low-achieving secondary students

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Introduction

Motivation

> *Achievement is not equitably spread throughout society; children from less affluent homes do disproportionately worse than those bought up in relative affluence. [. . .] Much research has attempted to articulate this relationship, whilst much more has ignored it, through denial, or the misguided belief that by supporting the affluent all will benefit through the 'trickle down' principle. (Gates, 2015)*

- Research with low achieving students for low achieving students.

Context

- Students from three low sets in an underperforming secondary school in East Midlands;
- Teachers were investigated by Rita Santos Guimaraes[1]
- Topic: Addition and subtraction of fractions.

[1] F12 - Investigating teachers' changes in practice with low-achieving students

Premises

- Build the knowledge on visual representations;
- Respect school practices.

Visual representations. Why?

- Issues with prior knowledge;
- Capitalize on the *intuitiveness* of visual representations;
- Reduce the reliance on verbal language;
- Promote visual skills.

Respect school practices. Why?

- Teachers have valuable tacit knowledge on their students;
- Bring in innovation without too much risk (Guimaraes, 2015);

The lessons

The lesson plans

- 12 lessons plans grouped into 3 packs (one per term);
- Worksheets + Comments for the teacher + cut-outs;
- Topics covered:
  - equivalent fractions,
  - comparison of fractions,
  - addition and subtraction,
  - mixed numbers and improper fractions,
  - word problems.

Three design principles

1. The lessons should enable students to build their knowledge about fractions on visual representations;
2. Students should have opportunities to solve the tasks without being told how to do it beforehand;
3. Keep the lesson plans coherent with participant teachers' current practices.

One single model

> *it seems wise to use the minimum number of representations consistent with the pedagogical function of the system. In many cases it may not be appropriate to use [multiple external representations] at all, since one representation may be sufficient and will minimise the split attention affect (Ainsworth, 2006)*

- Rectangular area model (RAM). Why?
  - Enables composition, decomposition and overlapping;
  - Self-similarity;
  - Truly bidimensional (one dimension for each operand).

Cornerstone lesson

- Emerged during the pilot study;
- Originally designed by one of the teachers;
- Discussed several times with the research team;
- Main features:
  - Not a lot of talking (video);
  - Central idea: adding fractions up to 1;

Outline of the lessons (1)

You can download the second version of all the lesson plans at barichello.coffee/bcme2018.

- Pack 1:
  - Lesson 1.1: introduction of rectangular area model,
  - Lesson 1.2: fractions (1/2, 1/4, 1/8 and 1/16),
  - Lesson 1.3: fractions (1/3, 1/6, 1/9 and 1/18),
  - Lesson 1.4: new denominators,
  - Lesson 1.5: diagrams.

Outline of the lessons (2)

- Pack 2
  - Lesson 2.1: Adding up to 1,
  - Lesson 2.2: Adding 1/2, 1/4, 1/8 and 1/16,
  - Lesson 2.3: Adding 1/3, 1/6, 1/9 and 1/18, 1/5, 1/10 and 1/15.
  - Word problems
- Pack 3
  - Lesson 3.1: Adding any two fractions using *cut-outs*,
  - Lesson 3.2: Adding any two fractions using diagrams,
  - Lesson 3.3: Word problems.

My data collection

Context

- Three groups:
  - Year 8 set 5 (out of 5),
  - Year 8 set 4 (out of 5),
  - Year 9 set 6 (out of 6).
- A bit more than 12 lessons each.

The students

- "working class area"
- Only one student was able to work out $\frac{3}{8} + \frac{2}{8}$ and none were able to work out $\frac{1}{10} + \frac{3}{5}$ and $\frac{1}{3} + \frac{1}{4}$ before the lessons;
- Invisible students (Pye, 1988);
- Low confidence and agency.

Data

- Main data:
  - Worksheets,
  - Within-class clinical interviews.
- Secondary data:
  - Informal talks with the teachers,
  - Meetings.

Data analysis and conclusion

Five issues

1. Interference of visual abilities in how students engaged with some tasks,
2. Emergence of reasoning anchored on the visual representations,
3. Ocurrence of generative reasoning,
4. Ups and downs related to the multiplicative aspect of fractions,
5. Ocurrence (or not!) of whole number bias and other rote procedures.

1 - Interference of visual abilities

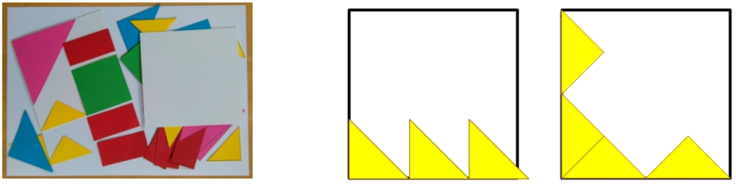

Figure 1: Rotating cut-outs was very demanding

2 - Reasoning and visual representations

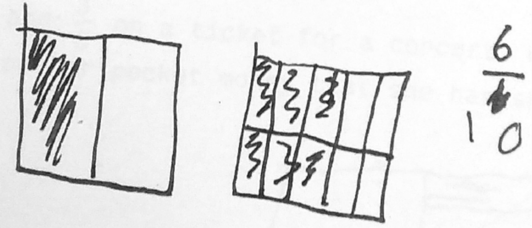

Figure 2: Answer to the question What is bigger, one half or six teenths?

3 - Generative reasoning

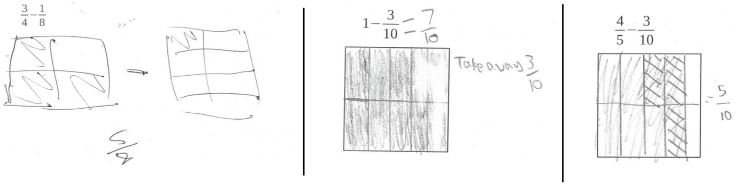

Figure 3: Three different representations for subtraction: two diagrams, erasing and crossing.

4 - The multiplicative aspects of fractions

- Visual representations lowered the threeshold in terms of multiplication and division prior knowledge, and...
- Several students showed signs of moving from counting, to counting in blocks to multiplication, but...
- Some students did not grasp the multiplciative aspect of fractions.

5 - Whole number bias

- Gone!
- Just a few mistakes related to the whole number bias;
- Some “doubling”, but only when questions were conducive.

Why?

- Meaning beyond symbolic.

Final example

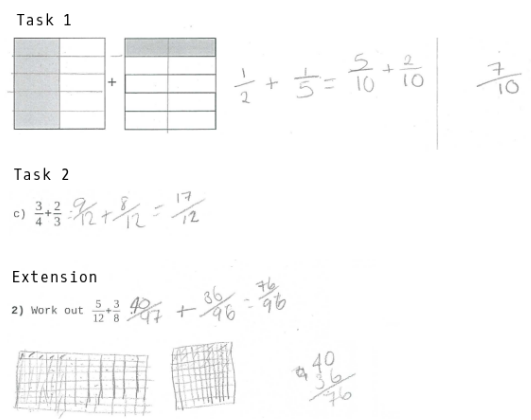

Figure 4: Three solutions from the same student

Major conclusions

- Reasoning and visual representations:
  - RAM as a grounding metaphor (Lakoff & Núñez, 2000)
    - Reasoning,
    - Generativity.
- Limitations:
  - Rotation (visual skills),
  - Multiplicative aspect.
  - A new and improved version of the lessons!

Extending the results

- Visual representations can work as basis to develop mathematical knowledge;
- Visual representations help communication and vocabulary acquisition;
- Visual skills are challenging and should not be taken for granted.

For a more visual pedagogy!

1. Choose representations carefully and teach not only how to read it, but how to operate on it;
2. Build the topic on the visual representations and not the other way around;
3. Accept visual arguments;
4. Pay attention to visual skills.

References

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Thank you!

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http://barichello.coffee/bcme2018